

The Lamb shift in de Sitter spacetime

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Abstract

We study the Lamb shift of both freely-falling and static two-level atoms in interaction with quantized conformally coupled massless scalar fields in the de Sitter-invariant vacuum. We find that the Lamb shifts of both freely-falling and static atoms are in structural similarity to that of an inertial atom immersed in a thermal bath in a Minkowski spacetime. For the freely-falling atom, the Lamb shift gets a correction as if it was immersed in a thermal bath at the Gibbons-Hawking temperature, thus revealing clearly the intrinsic thermal nature of de Sitter spacetime. For the static atom, the Lamb shift is affected by a combination of the effect of the intrinsic thermal nature of de Sitter spacetime and the Unruh effect associated with the inherent acceleration of the atom.

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I. INTRODUCTION

The Lamb shift is one of the most remarkable observable phenomena in physics which has been precisely measured in experiment and it has attracted a great deal of interest since its discovery in 1947 [1]. So far, the Lamb shift has been investigated in various circumstances, in the presence of cavities [2], or in a thermal bath [3–5], for example. It has recently been shown that the non-inertial motion of the atom also induces corrections to the Lamb shift [6–9]. However, all the aforementioned studies are concerned with flat spacetimes. Therefore, it remains interesting to see what happens if the atom is placed in a curved spacetime rather than a flat one. In this regard, it is interesting to note that the Lamb shift has recently been calculated of static atoms in interaction with fluctuating vacuum massless scalar fields in a curved background, i.e., the exterior of Schwarzschild black hole, and it is found that the Lamb shift gets corrected as a result of both scattering of vacuum field modes off the spacetime curvature and the Hawking radiation from the black hole [10].

In the present paper, we plan to calculate the Lamb shift in a de Sitter spacetime which describes an empty universe with a positive cosmological constant. It is the unique maximally symmetric curved spacetime and enjoys an important status in curved spacetimes just as that of the Minkowski spacetime in flat spacetimes. Our interest in this issue are twofold. First, our universe is believed, according to the current observations and the inflation theory, to approach de Sitter spacetime in the far past and the far future. Second, there may exist a holographic duality between quantum gravity on de Sitter spacetime and a conformal field theory living on the boundary identified with the timelike infinity of de Sitter spacetime [11]. So, it is interesting to investigate the Lamb shift in this special curved spacetime and this is exactly what we plan to do in the present paper. Using an elegant formalism suggested by Dalibard, Dupont-Roc and Cohen-Tannoudji(DDC) [12, 13], which allows a separation of the contributions of vacuum fluctuations and the radiation reaction to the energy shifts, we will calculate the Lamb shift of both a freely falling atom and a static one with an inherent acceleration in interaction with vacuum fluctuations of quantized massless conformally coupled scalar fields in de Sitter spacetime. Let us note that the quantization of scalar fields in this spacetime has been extensively studied in the literature [14–23].

When referring to the vacuum fluctuations in quantum field theory, we should first specify the vacuum states. Our research is done in the de Sitter-invariant vacuum state which is deemed to a natural vacuum in this spacetime since it preserves the de Sitter invariance [20] and therefore enjoys a special status as that of the Minkowski vacuum in flat spacetimes. Our calculations show that the Lamb shift for an atom moving on a timelike geodesic (freely falling) is identical to that of an inertial one immersed in a thermal bath at the Gibbons-Hawking temperature. For a static atom, the Lamb shift is modified as opposed to that in a flat spacetime by the combined effects of the intrinsic thermal nature of de Sitter spacetime characterized by the Gibbons-Hawking temperature and the atomic inherent acceleration.

II. THE GENERAL FORMALISM

We consider a pointlike two-level atom interacting with quantized conformally coupled massless scalar field in de Sitter spacetime. The two stationary atomic eigenstates are represented by $|+\rangle$ and $|-\rangle$ and their corresponding energies are $\frac{1}{2}\omega_0$ and $-\frac{1}{2}\omega_0$ respectively. The Hamiltonian that determines the evolution of the atom-field system with respect to the proper time of the atom, τ , is given by,

$$H = H_A(\tau) + H_F(\tau) + H_I(\tau) . \quad (1)$$

Here $H_A(\tau)$ is the Hamiltonian of the atom. In Dicke's notation [24], it is given by

$$H_A(\tau) = \omega_0 R_3(\tau) \quad (2)$$

with $R_3(0) = \frac{1}{2}|+\rangle\langle+| - \frac{1}{2}|-\rangle\langle-|$; $H_F(\tau)$ is the free Hamiltonian of the quantum scalar field

$$H_F(\tau) = \int d^3k \, \omega_{\vec{k}} \, a_{\vec{k}}^+ a_{\vec{k}} \frac{dt}{d\tau} , \quad (3)$$

in which $a_{\vec{k}}^+$ and $a_{\vec{k}}$ are the creation and annihilation operators with momentum \vec{k} ; $H_I(\tau)$ is the Hamiltonian that describes the interaction between the atom and the field,

$$H_I(\tau) = \mu R_2(\tau) \phi(x(\tau)) , \quad (4)$$

where μ is a small coupling constant that is assumed to be small, $R_2(0) = \frac{1}{2}i[R_-(0) - R_+(0)]$, and $R_+(0) = |+\rangle\langle-|$, and $R_-(0) = |-\rangle\langle+|$ are the atomic raising and lowering operators. These operators obey the angular momentum algebra: $[R_3, R_{\pm}] = \pm R_{\pm}$, $[R_+, R_-] = 2R_3$. $\phi(x)$ is the scalar field operator in de Sitter spacetime and it satisfies the wave equation

$$(\nabla_\mu \nabla^\mu + m^2 + \xi R) \phi(x) = 0 , \quad (5)$$

where m is the mass of the scalar field, ξ is the coupling constant, and R is the Ricci scalar curvature of the spacetime. In the case of conformally coupled scalar field in a four dimensional spacetime here, $\xi = 1/6$. The coupling is effective only on the atomic trajectory $x(\tau)$.

From the above Hamiltonians, we can derive the Heisenberg equations of the atomic and the field's variables and their solutions can then be divided into two parts: a free part which exists even when there is no coupling between the atom and the field; a source part that is caused by the coupling between the two and is characterized by the small coupling constant μ . Assuming the original state of the field is the de Sitter-invariant vacuum state, i.e., the Euclidean or Bunch-Davies vacuum state [15], choosing a symmetric operator ordering between the atomic and the field's variables, and proceeding in a manner similar to that in Ref. [6, 13], we can identify the following effective Hamiltonians to the order μ^2 ,

$$H_{vf}^{eff}(\tau) = \frac{1}{2}i\mu^2 \int_{\tau_0}^{\tau} d\tau' C^F(x(\tau), x(\tau')) [R_2^f(\tau'), R_2^f(\tau)] , \quad (6)$$

$$H_{rr}^{eff}(\tau) = -\frac{1}{2}i\mu^2 \int_{\tau_0}^{\tau} d\tau' \chi^F(x(\tau), x(\tau')) \{R_2^f(\tau'), R_2^f(\tau)\} , \quad (7)$$

the sum of which governs the time evolution of the atomic observables. Here $\{ , \}$ and $[,]$ denote the commutator and anticommutator respectively. $C^F(x(\tau), x(\tau'))$ and $\chi^F(x(\tau), x(\tau'))$ are separately the symmetric correlation function and the linear susceptibility of the field, and they are defined as

$$C^F(x(\tau), x(\tau')) = \frac{1}{2} \langle 0 | \{ \phi^f(x(\tau)), \phi^f(x(\tau')) \} | 0 \rangle , \quad (8)$$

$$\chi^F(x(\tau), x(\tau')) = \frac{1}{2} \langle 0 | [\phi^f(x(\tau)), \phi^f(x(\tau'))] | 0 \rangle . \quad (9)$$

Taking the expectation values of Eqs. (6) and (7) on a generic atomic state $|b\rangle$ yields the contributions of vacuum fluctuations and the radiation reaction to the energy shift of level b

$$(\delta E_b)_{vf} = -i\mu^2 \int_{\tau_0}^{\tau} d\tau' C^F(x(\tau), x(\tau')) \chi_b^A(\tau, \tau') , \quad (10)$$

$$(\delta E_b)_{rr} = -i\mu^2 \int_{\tau_0}^{\tau} d\tau' \chi^F(x(\tau), x(\tau')) C_b^A(\tau, \tau') , \quad (11)$$

in which $C_b^A(\tau, \tau')$ and $\chi_b^A(\tau, \tau')$ are, separately, the symmetric correlation function and the linear susceptibility of the atom. They are defined as

$$C_b^A(\tau, \tau') = \frac{1}{2} \langle b | \{ R_2^f(\tau), R_2^f(\tau') \} | b \rangle , \quad (12)$$

$$\chi_b^A(\tau, \tau') = \frac{1}{2} \langle b | [R_2^f(\tau), R_2^f(\tau')] | b \rangle . \quad (13)$$

They do not depend on the atomic trajectory and are determined by the internal structure of the atom itself. Their explicit forms are

$$C_b^A(\tau, \tau') = \frac{1}{2} \sum_d |\langle b | R_2(0) | d \rangle|^2 (e^{i\omega_{bd}\Delta\tau} + e^{-i\omega_{bd}\Delta\tau}) , \quad (14)$$

$$\chi_b^A(\tau, \tau') = \frac{1}{2} \sum_d |\langle b | R_2(0) | d \rangle|^2 (e^{i\omega_{bd}\Delta\tau} - e^{-i\omega_{bd}\Delta\tau}) . \quad (15)$$

Here $\omega_{bd} = \omega_b - \omega_d$ and the sum extends over a complete set of the atomic eigenstates.

III. THE LAMB SHIFT OF A FREELY FALLING ATOM IN DE SITTER SPACE-TIME

In this section, we consider the Lamb shift of a freely-falling atom in interaction with a quantized conformally coupled massless scalar field in de Sitter spacetime. There are several different coordinate systems that can be chosen to parameterize de Sitter spacetime [14]. Here we choose to work with the global coordinate system (t, χ, θ, ϕ) under which the freely-falling atom is comoving with the expansion. The line element is

$$ds^2 = dt^2 - \alpha^2 \cosh^2(t/\alpha) [d\chi^2 + \sin^2 \chi (d\theta^2 + \sin^2 \theta d\varphi^2)] \quad (16)$$

with $\alpha = 3^{1/2}\Lambda^{-1/2}$, where Λ is the cosmological constant. The parameter t is often called the world or cosmic time. The scalar curvature of the spacetime is $R = 12\alpha^{-2}$. The canonical quantization of a massive scalar field with this metric has been done in Ref. [15, 18, 20, 21]. In the global coordinates, solve the equation of motion to get a complete set of eigenmodes and define a de Sitter-invariant vacuum, then the Wightman function of the massive scalar field can be found [21]

$$G^+(x(\tau), x(\tau')) = -\frac{1}{16\pi\alpha^2} \frac{\frac{1}{4} - \nu^2}{\cos(\pi\nu)} F\left(\frac{3}{2} + \nu, \frac{3}{2} - \nu; 2; \frac{1 - Z(x, x')}{2}\right) \quad (17)$$

with F being the hypergeometric function and

$$\begin{aligned} Z(x, x') &= \sinh \frac{t}{\alpha} \sinh \frac{t'}{\alpha} - \cosh \frac{t}{\alpha} \cosh \frac{t'}{\alpha} \cos \Omega, \\ \cos \Omega &= \cos \chi \cos \chi' + \sin \chi \sin \chi' [\cos \theta \cos \theta' + \sin \theta \sin \theta' \cos(\varphi - \varphi')], \\ \nu &= \sqrt{\frac{9}{4} - \frac{12}{R}(m^2 + \xi R)}. \end{aligned} \quad (18)$$

In the massless and conformal coupling limit, the Wightman function for a freely-falling atom can be simplified to be

$$G^+(x(\tau), x(\tau')) = -\frac{1}{16\pi^2\alpha^2 \sinh^2(\frac{\tau - \tau'}{2\alpha} - i\epsilon)}. \quad (19)$$

This leads to the following two statistical functions of the field

$$C^F(x(\tau), x(\tau')) = -\frac{1}{32\pi^2\alpha^2} \left[\frac{1}{\sinh^2(\frac{\tau - \tau'}{2\alpha} - i\epsilon)} + \frac{1}{\sinh^2(\frac{\tau - \tau'}{2\alpha} + i\epsilon)} \right], \quad (20)$$

$$\chi^F(x(\tau), x(\tau')) = \frac{i}{4\pi \cos(\frac{i(\tau - \tau')}{2\alpha})} \delta'(\tau - \tau'). \quad (21)$$

Inserting them into Eqs. (10) and (11) and assuming the proper time interval $\Delta\tau = \tau - \tau'$ to be sufficiently long, we can calculate the contributions of vacuum fluctuations and the radiation reaction to the energy shift of level b . Here, it is not easy to calculate the involved integrals using the residual theorem and contour integration techniques. So, we first perform the Fourier transform on the above two statistical functions to obtain

$$C^F(x(\tau), x(\tau')) = \frac{1}{8\pi^2} \int_0^\infty d\omega \, \omega \coth(\pi\alpha\omega) (e^{i\omega\Delta\tau} + e^{-i\omega\Delta\tau}), \quad (22)$$

$$\chi^F(x(\tau), x(\tau')) = -\frac{1}{8\pi^2} \int_0^\infty d\omega \, \omega (e^{i\omega\Delta\tau} - e^{-i\omega\Delta\tau}). \quad (23)$$

Then the contributions of vacuum fluctuations and the radiation reaction can be expressed respectively as

$$\begin{aligned} (\delta E_b)_{vf} &= -\frac{i\mu^2}{16\pi^2} \sum_d |\langle b | R_2(0) | d \rangle|^2 \times \\ &\quad \int_0^\infty d\omega \int_0^\infty d\Delta\tau \, \omega \coth(\pi\alpha\omega) (e^{i\omega\Delta\tau} + e^{-i\omega\Delta\tau}) (e^{i\omega_{bd}\Delta\tau} - e^{-i\omega_{bd}\Delta\tau}), \end{aligned} \quad (24)$$

and

$$(\delta E_b)_{rr} = \frac{i\mu^2}{16\pi^2} \sum_d |\langle b|R_2(0)|d\rangle|^2 \times \int_0^\infty d\omega \int_0^\infty d\Delta\tau \omega (e^{i\omega\Delta\tau} - e^{-i\omega\Delta\tau})(e^{i\omega_{bd}\Delta\tau} + e^{-i\omega_{bd}\Delta\tau}). \quad (25)$$

Further simplification gives

$$(\delta E_b)_{vf} = \frac{\mu^2}{8\pi^2} \sum_d |\langle b|R_2(0)|d\rangle|^2 \int_0^\infty d\omega P\left(\frac{\omega}{\omega + \omega_{bd}} - \frac{\omega}{\omega - \omega_{bd}}\right) \coth(\pi\alpha\omega), \quad (26)$$

$$(\delta E_b)_{rr} = -\frac{\mu^2}{8\pi^2} \sum_d |\langle b|R_2(0)|d\rangle|^2 \int_0^\infty d\omega P\left(\frac{\omega}{\omega + \omega_{bd}} + \frac{\omega}{\omega - \omega_{bd}}\right). \quad (27)$$

Here and after P denotes the Principal Value. So, for the contribution of vacuum fluctuations to the energy shifts of the two levels, we have

$$(\delta E_+)_{vf} = \frac{\mu^2}{32\pi^2} \int_0^\infty d\omega P\left(\frac{\omega}{\omega + \omega_0} - \frac{\omega}{\omega - \omega_0}\right) \coth(\pi\alpha\omega), \quad (28)$$

$$(\delta E_-)_{vf} = \frac{\mu^2}{32\pi^2} \int_0^\infty d\omega P\left(\frac{\omega}{\omega - \omega_0} - \frac{\omega}{\omega + \omega_0}\right) \coth(\pi\alpha\omega). \quad (29)$$

In the above computation, we have used the relation $\sum_d |\langle b|R_2(0)|d\rangle|^2 = 1/4$. The contribution of vacuum fluctuations to the shifts of two different levels differs only in sign. For the contribution of the radiation reaction, we find

$$(\delta E_+)_{rr} = (\delta E_-)_{rr} = -\frac{\mu^2}{32\pi^2} \int_0^\infty d\omega P\left(\frac{\omega}{\omega + \omega_0} + \frac{\omega}{\omega - \omega_0}\right). \quad (30)$$

Obviously, the radiation reaction contributes the same to each energy level's shift and it is equal to that in a four dimensional Minkowski spacetime.

Adding up the contributions of vacuum fluctuations and the radiation reaction, we obtain the total energy shifts,

$$\delta E_+ = \frac{\mu^2}{16\pi^2} \int_0^\infty d\omega \left[\frac{\omega}{\omega + \omega_0} \frac{1}{e^{2\pi\alpha\omega} - 1} - \frac{\omega}{\omega - \omega_0} \left(1 + \frac{1}{e^{2\pi\alpha\omega} - 1}\right) \right], \quad (31)$$

$$\delta E_- = \frac{\mu^2}{16\pi^2} \int_0^\infty d\omega \left[\frac{\omega}{\omega - \omega_0} \frac{1}{e^{2\pi\alpha\omega} - 1} - \frac{\omega}{\omega + \omega_0} \left(1 + \frac{1}{e^{2\pi\alpha\omega} - 1}\right) \right]. \quad (32)$$

The relative energy shift, i.e., the Lamb shift, is given by $\Delta = \delta E_+ - \delta E_-$, or by $\Delta = (\delta E_+)_{vf} - (\delta E_-)_{vf}$ directly as the radiation reaction contributes the same to each level's shift. So, the Lamb shift in de Sitter space is entirely caused by vacuum fluctuations and is found to be

$$\Delta = \Delta_0 + \Delta_{T_f} \quad (33)$$

with

$$\Delta_0 = \frac{\mu^2}{16\pi^2} \int_0^\infty d\omega \left(\frac{\omega}{\omega + \omega_0} - \frac{\omega}{\omega - \omega_0} \right), \quad (34)$$

$$\Delta_{T_f} = \frac{\mu^2}{8\pi^2} \int_0^\infty d\omega P \left(\frac{\omega}{\omega + \omega_0} - \frac{\omega}{\omega - \omega_0} \right) \frac{1}{e^{2\pi\alpha\omega} - 1}. \quad (35)$$

Here Δ_0 is just the Lamb shift of an inertial two-level atom in a free Minkowski spacetime with no boundaries. It is logarithmically divergent and this divergence is expected for a non-relativistic treatment as what we do here. However, we can remove the divergence by introducing a cutoff on the upper limit of the integration. A reasonable cutoff frequency, ω_{max} , was suggested by Bethe who took it to be m (in SI units, it is mc^2/\hbar) with m being the mass of electron [25, 26]. Noticeably, the divergence can also be removed, if one resorts to a fully relativistic approach [27, 28]. Note that in the second term, Δ_{T_f} , a thermal factor $(e^{2\pi\alpha\omega} - 1)^{-1}$ appears. This term is similar to the correction to that of an inertial atom immersed in a thermal bath in a Minkowski spacetime at the temperature $T_f = 1/2\pi\alpha$ [3–5]. So, for the freely-falling atom in de Sitter spacetime, the Lamb shift is revised, as opposed to that in a flat unbounded spacetime, by a thermal-like term as if it was immersed in a thermal bath at the temperature $T_f = 1/2\pi\alpha$, which is exactly the Gibbons-Hawking temperature. We recover, in terms of the Lamb shift, Gibbons and Hawking's result that reveals the thermal nature of de Sitter spacetime [29]. Finally, let us note that since our universe is presumably in a phase of accelerating expansion and may approach a de Sitter space in the future, one may wonder whether the correction to the Lamb shift due to the spacetime being de Sitter rather Minkowski is experimentally measurable. In this regard, it is worth noting that the cosmological constant is, according to current observations, very tiny, corresponding to a Gibbons-Hawking temperature of only $\sim 10^{-30}K$. So, the correction is insignificant and it is therefore unrealistic for any actual experimental measurement.

IV. THE LAMB SHIFT OF A STATIC ATOM IN DE SITTER SPACETIME

For a static atom, we choose to work in the static coordinate system in which the line element is

$$ds^2 = \left(1 - \frac{r^2}{\alpha^2}\right) d^2\tilde{t} - \left(1 - \frac{r^2}{\alpha^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2). \quad (36)$$

The metric possesses a coordinate singularity $r = \alpha$. The origin $r = 0$ corresponds to the position of the observer, so the singularity is the event horizon for him. Just like the Rindler wedge in a flat spacetime, the coordinates $(\tilde{t}, r, \theta, \varphi)$ only cover part of de Sitter spacetime. The worldlines of constant r are uniformly accelerated timelike curves and only $r = 0$ is a geodesic (with proper time t). The static and the global coordinates are related by

$$r = \alpha \cosh(t/\alpha) \sin \chi, \quad \tanh(\tilde{t}/\alpha) = \tanh(t/\alpha) \sec \chi. \quad (37)$$

Obviously, the worldline $r = 0$ in the static coordinate coincides with the worldline $\chi = 0$ in the global coordinate and an atom at rest with $r \neq 0$ in the static coordinate will be accelerated relative to the observer at rest in the global coordinate with $\chi = 0$.

Similarly, in the static coordinate system, one can find out a set of complete eigenmodes by solving the field equation [22, 23] and define a de Sitter-invariant vacuum. Then the Wightman function for the massless conformally coupled scalar field is [30, 31]

$$G^+(x(\tau), x(\tau')) = -\frac{1}{8\pi^2\alpha^2} \frac{\cosh(\frac{r^*}{\alpha}) \cosh(\frac{r'^*}{\alpha})}{\cosh(\frac{\tilde{t}-\tilde{t}'}{\alpha} - i\epsilon) - \cosh(\frac{r^*-r'^*}{\alpha})} \quad (38)$$

with $r^* = \frac{\alpha}{2} \ln \frac{\alpha+r}{\alpha-r}$. For a static atom, it can be simplified to be

$$G^+(x(\tau), x(\tau')) = -\frac{1}{16\pi^2\kappa^2 \sinh^2(\frac{\tau-\tau'}{2\kappa} - i\epsilon)} \quad (39)$$

with $\kappa = \sqrt{g_{00}} \alpha$. In deriving the above result, we have used the relation, $\Delta\tau = \sqrt{g_{00}} \Delta\tilde{t}$. Two statistical functions of the field are then easily obtained

$$C^F(x(\tau), x(\tau')) = -\frac{1}{32\pi^2\kappa^2} \left[\frac{1}{\sinh^2(\frac{\tau-\tau'}{2\kappa} - i\epsilon)} + \frac{1}{\sinh^2(\frac{\tau-\tau'}{2\kappa} + i\epsilon)} \right], \quad (40)$$

$$\chi^F(x(\tau), x(\tau')) = \frac{i}{4\pi \cos(\frac{i(\tau-\tau')}{2\kappa})} \delta'(\tau - \tau'). \quad (41)$$

Performing the Fourier transform on the statistical functions and then following the same procedure as that in the proceeding Section, we find that the Lamb shift for the static atom can be written as

$$\Delta = \Delta_0 + \Delta_{T_s} \quad (42)$$

with

$$\Delta_{T_s} = \frac{\mu^2}{8\pi^2} \int_0^\infty d\omega P \left(\frac{\omega}{\omega + \omega_0} - \frac{\omega}{\omega - \omega_0} \right) \frac{1}{e^{2\pi\kappa\omega} - 1}. \quad (43)$$

Here Δ_{T_s} is the same as the correction to the Lamb shift of an inertial atom induced by the presence of a thermal bath at the temperature

$$T_s = \frac{1}{2\pi\kappa} = \frac{1}{2\pi\alpha\sqrt{g_{00}}}. \quad (44)$$

in a Minkowski spacetime. This differs from what was obtained in the case of the freely-falling atom ($T_f = 1/2\pi\alpha$). Remarkably, the two are related by

$$T_s^2 = \left(\frac{1}{2\pi\alpha} \right)^2 + \left(\frac{a}{2\pi} \right)^2 = T_f^2 + T_U^2 \quad (45)$$

with

$$a = \frac{r}{\alpha^2} (1 - \frac{r^2}{\alpha^2})^{-1/2} \quad (46)$$

being the inherent acceleration of the static atom. The first term on the right hand side of Eq. (45) is the square of the Gibbons-Hawking temperature of de Sitter spacetime. The second term, which is acceleration-dependent, is a result of the Unruh effect. So, the correction to the Lamb shift of the static atom is a combined effect of the thermal nature of de Sitter spacetime characterized by the Gibbons-Hawking temperature and the Unruh effect which is associated with the inherent acceleration of the atom. Thus, in terms of the Lamb shift, the temperature felt by the static atom is the square root of the sum of the squared Gibbons-Hawking temperature and the squared Unruh temperature associated with its inherent acceleration. It is worthwhile to note that the same relation as that of Eq. (45) is also obtained in other different physical contexts [32–34].

V. SUMMARY

Using the DDC formalism, we have calculated the contributions of vacuum fluctuations and the radiation reaction to the energy level shifts of both freely-falling and static two-level atoms in interaction with a conformally coupled massless real scalar field in the de Sitter-invariant vacuum and obtained the Lamb shifts. The Lamb shifts of both the freely-falling and the static atoms are in structural similarity to that of an inertial atom immersed in a thermal bath in the Minkowski spacetime.

For a freely-falling atom, the Lamb shift gets a correction as if it was immersed in a thermal bath at the Gibbons-Hawking temperature $T_f = 1/2\pi\alpha$. This clearly reveals the intrinsic thermal nature of de Sitter spacetime. The correction is however insignificant in terms of actual experimental measurements, as the cosmological constant is very tiny, corresponding to an effective temperature as low as $\sim 10^{-30}K$.

For a static atom, the Lamb shift is affected by a combination of the Gibbons-Hawking effect of de Sitter spacetime and the Unruh effect associated with the inherent acceleration of the atom. In fact, the Lamb shift for a static atom in de Sitter spacetime is the same as that of an inertial atom in a thermal bath in the Minkowski spacetime at the temperature which is a square root of the sum of the squared Gibbons-Hawking temperature and the squared Unruh temperature associated with the atomic inherent acceleration.

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